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| **Chapter:** | 5 |

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**Setup**

*> Means.Noise <- noise\_means.df$MeanNoise*

*> Means.Distance <- noise\_means.df$Distance*

*> Means.N <- noise\_means.df$N*

*> Individuals.Noise <- noise\_individuals.df$Noise*

*> Individuals.Distance <- noise\_individuals.df$Distance*

**A) Why weighted regression?**

*> noise\_means.lm <- lm(Means.Noise ~ Means.Distance, weights=Means.N)*

Weighted regression is required here because each response is the mean noise for n\_i observations at distance d\_i.

Each of these means will have variance equal to population variance divided by the number of observations n\_i at distance d\_i. Since one of the assumptions for regression requires constant variance, we multiply each of the responses by their respective n\_i to scale them all to have the same variance. (Can also consider it as ascribing more weight to response variables made up of larger numbers of observations).

**B) Lack of Fit test (weighted model method)**

*> noise\_individuals.lm <- lm(Individuals.Noise ~ Individuals.Distance)*

It can be shown that the Residual Sum of Squares in the weighted regression model is equal to the Lack of Fit Sum of Squares in the unweighted regression model.

Also, it can be shown that the Residual Degrees of Freedom in the weighted model is equal to the Lack Of Fit Degrees of Freedom in the unweighted regression model.

The Residual Sum of Squares of the unweighted model can be partitioned into the Pure Error Sum of Squares + the Lack of Fit Sum of Squares.

**Using these facts to compute a Lack of Fit test statistic (alpha=0.05):**

*> ss.lof <- deviance(noise\_means.lm)*

*> df.lof <- noise\_means.lm$df.residual*

*> ss.pe <- deviance(noise\_individuals.lm) – ss.lof*

*> df.pe <- noise\_individuals.lm$df.residual – df.lof*

*> f.statistic <- (ss.lof/df.lof) / (ss.pe/df.pe)*

*> p.value <- pf(f.statistic, df.lof, df.pe, lower.tail=FALSE)*

f.statistic=5.07182, **p.value=0.0001249549 < 0.05 => there is a lack of fit.**

**C) Lack of Fit test (model-free ANOVA method)**

(alpha/level of significance still 0.05)

*> noise\_individuals.aov*

*<- aov(Individuals.Noise ~ factor(Individuals.Distance))*

*> anova(noise\_individuals.lm, noise\_individuals.aov)*

*Res.Df RSS Df Sum of Sq F Pr(>F)*

*1 116 551.23*

*2 110 429.57 6 121.66 5.1924 9.742e-05 \*\*\**

f.statistic = 5.1924, **p.value = 9.742e-05 < 0.05 => there is a lack of fit.**

**D) Why do the test statistics differ slightly (5.07 vs 5.19)?**

It is helpful to look at the manual computation behind the model-free analysis of variance method:

*> SD <- by(Individuals.Noise, factor(Individuals.Distance), sd)*

*> ss.pe <- sum((Means.N-1)\*SD^2) #using Means.N column for convenience only*

*> df.pe <- sum(Means.N-1)*

*> ss.lof <- deviance(noise\_individuals.lm) - ss.pe*

*> ss.df <- noise\_individuals.lm$df.residual – df.pe*

*> f.statistic <- (ss.lof/df.lof) / (ss.pe/df.pe)* **= 5.1924(28) same as ANOVA**

Both methods in B and C use the same RSS calculated from the unweighted linear model.

Method B first estimates the unweighted model’s Lack of Fit Sum of Squares and Lack of Fit Degrees of Freedom estimates using the weighted model’s RSS and RSS.df.

It uses these estimates and the relationship RSS = Pure Error + Lack of Fit to calculate the value of the Pure Error Sum of Squares and Degrees of Freedom.

Method C calculates first the model-free estimate for Pure Error Sum of Squares and Pure Error Degrees of Freedom using AOV/ANOVA.

It then uses this estimate and the relationship RSS = Pure Error + Lack of Fit to calculate the Lack of Fit Sum of Squares and Degrees of Freedom.

Since we are using a different estimate in each test to calculate our final f-statistic, it is not a surprise that for only 118 observations they would not produce exactly the same value in the end.

Especially when the Lack of Fit test is a very powerful test which is capable of picking up even minute differences in the models being tested.